Graphing the Derivative of a Function

**Warm-up:** Part 1 - What comes to mind when you think of the word 'derivative'?

Part 2 - Graph \( f(x) = -x^2 + 3 \). Then find \( f'(x) \) and graph it.

\[ f'(x) = -2x \]
What if you're not given the equation of the original function?

Example 1

Directions: Given the function on the left, graph its derivative on the right.

1) Graph of $f(x) = 2x - 5$  
Graph of $f'(x) = 2$
2) Graph of $f(x)$

First: Write the piecewise function that represents the graph above.

$$f(x) = \begin{cases} 
|x|, & X < 2 \\
-2x + 2, & X \geq 2
\end{cases}$$

Second: Find $f'(x)$ and graph it below.

$$f'(x) = \begin{cases} 
-1, & X < 0 \\
1, & 0 < X < 2 \\
-2, & X \geq 2
\end{cases}$$
3) Graph of $f(x) = x^2$

Graph of $f'(x) = 2x$
What about these graphs? It would be difficult to come up with the equations. Can you graph their derivatives?

At this time, go to this website on your iPads:
http://www.khanacademy.org/math/calculus/e/derivative_intuition
What do you notice about the answer column on the right and the graph?
Example 2: Estimate the slope of the tangent line at various $x$-values.

Now graph $f'(x)$ below.
Example 3

*Directions*: Given the function on the left, graph its derivative on the right.

What do you notice about the graph of $f(x)$ when $f'(x)$ is negative?

What do you notice about the graph of $f(x)$ when $f'(x)$ is positive?

What do you notice about the graph of $f(x)$ when $f'(x)$ is zero?
The sign of $f'$ tells us whether $f$ is increasing or decreasing.

<table>
<thead>
<tr>
<th>If $f'$ is...</th>
<th>...then $f$ is</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive ($f' &gt; 0$)</td>
<td>Increasing</td>
</tr>
<tr>
<td>Negative ($f' &lt; 0$)</td>
<td>Decreasing</td>
</tr>
<tr>
<td>Zero ($f' = 0$)</td>
<td>Maximum or Minimum</td>
</tr>
<tr>
<td></td>
<td>Usually</td>
</tr>
</tbody>
</table>
Example 4: Given the graph of $f(x)$, sketch the graph of $f'(x)$.

<table>
<thead>
<tr>
<th>$f(x)$</th>
<th>Interval</th>
<th>$f'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inc.</td>
<td>$(-\infty,-1)$</td>
<td>Pos.</td>
</tr>
<tr>
<td>max</td>
<td>$X=-1$</td>
<td>0</td>
</tr>
<tr>
<td>Dec.</td>
<td>$(-1,1)$</td>
<td>Neg.</td>
</tr>
<tr>
<td>min</td>
<td>$X=1$</td>
<td>0</td>
</tr>
<tr>
<td>Inc.</td>
<td>$(1,\infty)$</td>
<td>Pos.</td>
</tr>
</tbody>
</table>
Example 5: Given the graph of $f(x)$, sketch the graph of $f'(x)$. 
Example 6: Multiple Choice

Which graph on the right is the derivative graph of the function below on the left?

\[
\begin{array}{c|c|c}
\text{Interval} & f(x) & f'(x) \\
\hline
-6 & \max & 0 \\
(-6, -1) & \text{Dec.} & \text{Neg} \\
-1 & \min & 0 \\
\end{array}
\]
Example 7: Multiple Choice

Which graph on the right is the derivative graph of the function below on the left?

A.

B.

C.
Graphing a Function Given its Derivative Graph

**Directions:** The function on the left is $f'(x)$. Graph $f(x)$ on the right.

What type of function would $f(x)$ be? *Cubic*

Graph of $f'(x) = x^2$

Graph of $f(x) = \frac{1}{3}x^3 + C$
Example 8

**Directions:** The function on the left is \( f'(x) \). Graph \( f(x) \) on the right.

<table>
<thead>
<tr>
<th>( f'(x) )</th>
<th>Interval</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neg</td>
<td>(( -\infty, -4))</td>
<td>Dec. min.</td>
</tr>
<tr>
<td>0</td>
<td>( x = -4 )</td>
<td></td>
</tr>
<tr>
<td>Pos</td>
<td>(( -4, -\frac{1}{2}))</td>
<td>Inc. max.</td>
</tr>
<tr>
<td>0</td>
<td>( x = -\frac{1}{2} )</td>
<td></td>
</tr>
<tr>
<td>Neg</td>
<td>(( -\frac{1}{2}, 4))</td>
<td>Dec. min.</td>
</tr>
<tr>
<td>0</td>
<td>( x = 4 )</td>
<td></td>
</tr>
<tr>
<td>Pos</td>
<td>(( 4, \infty))</td>
<td>Inc.</td>
</tr>
</tbody>
</table>

Graph of \( f'(x) \)

Graph of \( f(x) \)
Example 9: Given the graph of the derivative, sketch a possible graph for the function.
Example 10: Multiple Choice

The function on the left is $f'(x)$. Which graph on the right is $f(x)$?

<table>
<thead>
<tr>
<th>$f'(x)$</th>
<th>Int.</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neg.</td>
<td>(-∞, -4)</td>
<td>Dec.</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Pos.</td>
<td>(-4, 2)</td>
<td>Inc.</td>
</tr>
</tbody>
</table>

A. [Graph A]
B. [Graph B]
C. [Graph C]
Example 11: Multiple Choice

The function on the left is $f'(x)$. Which graph on the right is $f(x)$?

A. 

B. 

C. 

[Graphs A, B, and C are shown, with C circled to select the correct answer.]
Recalling the Lesson: Fill in the blank.

1. $f'(x)$ is zero when $f(x)$ is \( \max/\min \).

2. $f(x)$ is increasing when $f'(x)$ is \( \text{pos.} \).

3. $f'(x)$ is negative when $f(x)$ is \( \text{decreasing} \).
Use these websites to practice...

Practice graphing a derivative given the graph of the original function:
http://webspace.ship.edu/msrenault/GeoGebraCalculus/derivative_try_to_graph.html

Practice graphing an original function given a derivative graph:
http://webspace.ship.edu/msrenault/GeoGebraCalculus/derivative_app_1_graph_AD.html

Multiple Choice: Graphing a derivative.
http://webspace.ship.edu/msrenault/GeoGebraCalculus/derivative_matching.html

Multiple Choice: Graphing an original function given a derivative.
http://webspace.ship.edu/msrenault/GeoGebraCalculus/derivative_matching_antiderivative.html